

Numerical ranges in a strip

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Abstract

It is shown that a matrix is power bounded if and only if the numerical range of all its powers lies in a fixed strip of the complex plane. In particular, a matrix A is power bounded if and only if the sums $A^k + A^{*k}$ are uniformly bounded with respect to $k = 1, 2, \dots$. A number of analogous splittings are also obtained, especially for the Cesàro means of powers. The method is based on the concept of numerical range, the Schur triangularization of matrices, and a recent inequality of Kittaneh. The extension of this result to linear operators on a Hilbert space is given. Analogous problems in the Gerschgorin set setting are also investigated.