Incomplete alternating projection method for large inconsistent linear systems

Agnieszka Suchocka

University of Zielona Góra, Poland

Abstract

Let A be an $n \times m$ real matrix and let $b \in \mathbb{R}^m$. We consider the problem of finding an approximative solution $x \in \mathbb{R}^n$ of a large scale system of linear equations

$$A^\top x = b,$$

if such a solution exists. In practice these systems are often inconsistent.

The problem is a special case of the following problem: Let $\mathcal{P}, \mathcal{Q}$ be nonempty and affine subspaces. In practice we want to find an element of the intersection $\mathcal{P} \cap \mathcal{Q}$ or find points $p \in \mathcal{P}$ and $q \in \mathcal{Q}$ which realize the distance between these two subspaces. In order to solve the problem we deal with a modification of the so called *alternating projection method* (APM). APM generate a sequence (x_k) by the following iterative scheme:

$$x_{k+1} = P_{\mathcal{P}} P_{\mathcal{Q}} x_k.$$

We take in the modification an approximative projection $\tilde{P}_{\mathcal{P}}$ instead of an exact projection $P_{\mathcal{P}}$ with appropriate stopping criteria. We modify the APM in such a way that the Fejér monotonicity with respect to $\mathrm{Fix} P_{\mathcal{P}} P_{\mathcal{Q}}$ and the convergence of (x_k) to an element of $\mathrm{Fix} P_{\mathcal{P}} P_{\mathcal{Q}}$ is preserved.

We present preliminary numerical results for our method.

Keywords

Alternating projection method, Fejér monotonicity, Approximative projection, Residual projection.

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