The positive realization problem in normal transfer matrices with simple real poles*

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Abstract

The positive realization problem is formulated as follows: Let $H(z) \in \mathbb{R}^{r\times s}(z)$ be a transfer matrix, it is said to admit a positive realization (A, B, C) if we find nonnegative matrices $A \in \mathbb{R}^{N\times N}$, $B \in \mathbb{R}^{N\times s}$, $C \in \mathbb{R}^{r\times N}$ and $D \in \mathbb{R}^{r\times s}$ such that $H(z) = C[zI - A]^{-1}B + D$. Moreover, this realization will be minimal if it has the minimal dimension.

It has been studied for several authors although, they are mainly concentrated on single-input single-output (SISO) systems. The positive realization problem for multi-input multi-output (MIMO) linear systems is less studied. For instance, K. H. Forster and B. Nagy studied the nonnegative realizability of transfer matrices of discrete-time systems with a primitive state matrix. And in continuous-time systems, R. Cantó, B. Ricarte and A. M. Urbano computed a procedure to obtain a minimal positive realization (A, B, C, D) with the matrix A in the Jordan form.

In this work we deal with MIMO systems and solve the positive realization problem of certain *normal transfer matrices* with simple real poles. Moreover, we give necessary and sufficient conditions to obtain a minimal positive realization.

The concept of *normal rational matrix* was introduced by Lampe and Rosenwasser in multivariable linear control theory with the purpose to advance in the realization problem. Given a transfer matrix expressed by its standard rational form $H(z) = \frac{M(z)}{d(z)}$, where $M(z) \in \mathbb{R}^{r\times s}[z]$ is a polynomial matrix and $d(z)$ is an scalar polynomial (the minimal common denominator), they define it as a *normal transfer matrix* if any second order minor of $M(z)$ is zero or is divisible by $d(z)$.

Later, it has been proved that in transfer matrices, *normal transfer matrix* is equivalent to *irreducible transfer matrix*, which means that for any root z_i of $d(z) = 0$, the matrix $M(z_i)$ is not equal to the $r \times s$ zero matrix ($M(z_i) \neq 0$). Therefore, the dimension N of any minimal realization (A, B, C, D) will be equal to the order n of the transfer matrix $H(z)$.

Keywords

Minimal realizations, Multivariable systems, Transfer matrices, Positive realizations.

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