On using generalized Cramér-Rao inequality to REML estimation in linear models

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Abstract

The main aim of considerations in the problem of estimation of variance components σ_1^2 and σ_2^2 by using the ML-method and REML-method in normal mixed linear model $N\{Y, E(Y) = X\beta, Cov(Y) = \sigma_1^2 V + \sigma_2^2 I_n\}$ was concerned in the examination of theirs efficiency. It is particularly important when an explicit form of these estimators is unknown and we search for the solutions of the likelihood equations system by using different iterative procedures e.g.: Newton-Raphson method, Fisher's method of scoring, EM algorithm, Monte Carlo methods, gradient procedure (see papers: [4], [5], [6], [8]). For the sake of statistical properties of obtained solutions, unusually useful, are the numerical procedures which allow to control the biases and variances of these estimators. Some approximations of these parameters obtained without simulations for ML- and REML-estimators of variance components are presented in the paper [2]. Residual maximum likelihood estimation is often preferred to maximum likelihood estimation as a method of estimating covariance parameters in linear models because it takes account of the loss of degrees of freedom in estimating the mean and produces unbiased estimating equations for the variance parameters (cf. papers [2], [7]). Since the REML-estimators are "almost unbiased" for variance components σ_1^2 and σ_2^2 we may apply the generalized Cramér-Rao inequality to determine upper bounds of variances of the estimators (see [3]). This fact can be used to making alternative iterative procedures giving good solutions of the likelihood equations system (cf. [1], [2]). In the paper the numerical calculations for unbalanced random models corresponding to one-way layouts are also given.

Keywords

Mixed linear models, ML-and REML estimation, Variance components, Fishers information, Cramér-Rao inequality, Iterative MIVQUE.

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