Loewner comparability of moment matrices in cubic mixture models

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Abstract

Mixture experiments are experiments in which the experimental conditions are the relative proportions of ingredients of a whole. Given a second-degree polynomial regression model for such experiments, Draper and Pukelsheim (1999) and Draper, Heiligers, and Pukelsheim (2000) have identified a class of designs (the class of so-called weighted centroid designs) that is essentially complete with respect to many popular design criteria, that is, for any given design there is a weighted centroid design that is at least as good as the given design relative to the given design criterion. The crucial steps towards this result are: (a) understanding the inherent symmetry of moment matrices in this model (invariance with respect to a subgroup of permutations), and (b) a characterization of Loewner comparability of moment matrices.

One natural question is whether a similar completeness result can be reproduced for cubic mixture models. Mikaeili's (1993) result on D-optimal designs for a cubic mixture model shows that the class of weighted centroid designs cannot be essentially complete in this case. Instead, his result gives rise to the conjecture that the class of weighted centroid designs needs to be augmented in order to obtain an essentially complete class.

As in the quadratic case, understanding symmetry and Loewner comparability of moment matrices is essential. I will show how Andersson's (1975) result on variance-covariance matrices of invariant distributions can be put to use here, and I will discuss the completeness issue for mixture experiments with a small number of ingredients.

References:

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